**Multivariable Calculus – Spring 2016**

**Test 1-Answers**

**Part 1**

1. Give the parametric equations of the line through the point in the direction of the vector .
2. Give the equation of the plane containing the point and the line found in part A.

First we note that the vector is on the plane. We take as the normal vector . So, the equation of the plane is

1. Which point on the line you found in Part A is closest to ?

If the answer this question is the point , we see that

We find by “adding” this vector to . This yields .

**Part 2**

Let be the graph of and, let be a vector function describing the space curve .

1. At what point does the curve meet the surface ?

To find out when the curve meets the surface, we need to simultaneously solve the equations:

We do this by plugging the first three equations into the fourth. This yields:

So, the point where they meet occurs when . This is the point .

1. Give the equation of the plane tangent to at the point you found in part A.

The equation of the plane tangent to a graph is given by:

Here, and . The partial derivatives are and . We see that and . So, the equation of the tangent plane is:

1. Give the parametric form of the line tangent to at the point .

We see that goes through when . So, the tangent line follows the vector . To find this vector, we find . So . Therefore, the parametric form of the line is:

1. Is the line you found in part C on the plane you found in part B? Explain.

The vector normal to the plane is: and the line goes along the vector . Since we see that the vector for the line is not orthogonal to the normal vector for the plane. Therefore the line is not on the plane.

Alternatively, we could point out that the point which is on the line does not satisfy the equation of the plane, so the line cannot lie on the plane.

**Part 3**

1. Let be a function of three variables. Give the limit definition of the directional derivative where vector is a unit vector.
2. The depth, in meters, of a lake is given by . There is a goat floating at position (-1,1) and a rock at position (3,-2). As the goat begins swimming toward the rock, does the lake get deeper or shallower? Explain.

This question is asking for the derivative of in the direction of the vector between the goat and the rock. This vector is: . The unit vector in this direction is: . The gradient of is given by: . At the point for the goat, the gradient is: . So, the directional derivative is given by: . So, the water is getting deeper.

1. There is a babirusa swimming at position . In which direction does the water get shallower the most for the babirusa?

We see that . This is the direction in which the water gets deepest. The direction in which it gets the most shallow is the opposite this vector; i.e.,.

**Part 4**

1. Assume is a function of the three variables and , each of which is a function of . Write down the chain rule formula for the derivative of with respect to in this case.
2. Imagine that the temperature, measured in degrees Celsius, at the point is given by . A butterfly is moving through space so that its position at time , measured in seconds, is given by the vector function . Use the multivariable chain rule to determine whether the butterfly getting hotter or cooler when it passes through the point ? Explain.

We see that the butterfly goes through at time . We see that so that . That is to say, at the point , ; ; and .

Moreover, ;and . So, at the point we have: ; and .

We conclude that . Since this is positive, the butterfly is getting warmer.

**Part 5**

Let .

1. Is the point on a level set for this function? If so, give the corresponding level. If not, give a reason.

Every point of the domain is on some level set. The level corresponding to the level set containing is

1. What is the equation of the plane tangent to the level set of containing ?

The equation of the plane tangent to a level set at a point is given by the equation: . Here we have: ; and . So ; and . So, the equation of the tangent plane is:

1. Give a vector in the plane that is tangent at (4,2,0) to the horizontal trace of this level set.

The level set has the equation . The horizontal trace of this level set is the curve in the -plane we get by setting . That is: . We can find the slope of this curve with implicit differentiation: . At the point in question, we have . This is described by the vector . In 3-space, this is the vector .

Alternatively, the tangent plane we found in part B consists of all vectors that are tangent to some curve on the level set at the point . So, this question is asking for a vector which is on the tangent plane and on the plane . The normal vector for the tangent plane is: . The normal vector for the plane is . So, we take the cross product of these two normal vectors: